The problem of the difference between the kinetic energy of an electron in the constrained box obtained from the uncertainty relationship and that derived from the Schrödinger equation -- 2017.01.28 --

Let us determine the kinetic energy of electrons trapped in the wall of infinite potential by the Heisenberg's uncertainty relationship, $\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$.

Even in one dimension the conclusion may not change essentially, so we discuss this case. The kinetic energy (T) of an electron with mass m and velocity u is $(1/2)mu^2$. It is rewritten in terms of momentum ($p = mu$) and becomes $(1/2m)p^2$. Let us calculate the kinetic energy using "ambiguity of momentum" (Δp).

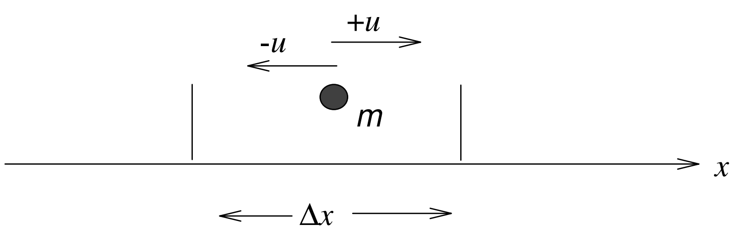

Fig. 1. The particle that moves within region Δx.

The range of movement is defined as Δx, and the momentum includes ambiguity Δp (that is, it is in the range of $p \pm \Delta p$ as p is the correct value). The mean value of the square of p ($\overline{p^2}$) is obtained by averaging the momentum including ambiguity. Since the particles cause rightward motion and leftward motion occur with the same probability, the momentum is averaged to 0 ($\overline{p} = 0$).

$$(\Delta p)^2 = (p - \overline{p})^2_{aver} = \overline{p^2}$$

Using the above relationship, the average energy ($\overline{E}$) is,

$$\overline{E} = \frac{1}{2m}(\Delta p)^2$$

This is put into the uncertainty relationship.

$$\Delta p \cdot \Delta x \geq \frac{\hbar}{2} \quad \Rightarrow \quad \Delta p \geq \frac{\hbar}{2\Delta x}$$

$$\overline{E} \geq \frac{\hbar^2}{8m(\Delta x)^2} \qquad 1$$

Next, we seek the energy of an electron in the same box from the Schrödinger equation. This is given by solving the next equation,

$$E\psi(x) = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x)$$

If the size of the box is set Δx , then,

$$E(n_x) = \left(n_x^2\right)\frac{h^2}{8m\Delta x^2} \qquad 2$$

is given.

Compare 1 and 2 equations. When $n_x = 1$, The energy by Eq.1 is small by the factor of $1/4\pi^2$.

Where does this difference come from?

Recently, Ozawa has shown that the uncertainty relationship so far is incomplete and submitted a new one (Ozawa's inequality). [1] The inequality is,

$$\Delta x \cdot \Delta p + \Delta x' \cdot \Delta p + \Delta x \cdot \Delta p' \geq \frac{\hbar}{2} \qquad 3$$

$\Delta x'$ and $\Delta p'$ are the intrinsic quantum fluctuation of particles. Since these terms in the left expression are not included, is Eq. 1 small? If so, the intrinsic quantum fluctuation may be derived from the difference between 1 and 2 equations.

1) Ozawa, Masanao (2003), "Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement", *Physical Review A* **67** (4).